Homework 2 - Due Wednesday October 11th

Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions

(1) Recall that the Stirling number of the second kind $S(n, k)$ are the number of partitions of an n-set into k blocks. Prove the following recurrance:

$$S(n, k) = kS(n - 1, k) + S(n - 1, k - 1)$$

(2) How many positive integers strictly less than 2102 are divisible by 2, 3, or 5?

(3) A *composition* of n with k parts is an ordered sum $x_1 + x_2 + \ldots + x_k = n$ such that each x_i is a positive integer. Give a bijection between compositions of n with k parts and subsets of $[n - 1]$ of size $k - 1$.

(4) Suppose we have a collection of subsets of $[n]$ such that each pair of subsets has at least one element in common. What is the maximum possible size of the collection?

(5) Let $n \geq 7$. Count the number of $0/1$ strings of lnegth n that contain at least $n - 3$ consecutive 1s.

(6) Problem 26 parts (a) and (b) of section 2.3 of your text.

(7) Prove that the number of onto (surjective) mappings from a set of size m to a set of size n is divisible by $n!$