Homework 1 - Due Wednesday October 4th

Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions

(1) Give a combinatorial proof of the following binomial coefficient identity:

$$\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$$

(2)(a) How many different necklaces can be formed from 6 different colored beads, where two necklaces are considered the same if one can be rotated to exactly match the other?

(b) How many different necklaces can be formed from n different colored beads?

(3) Number 19 Section 2.3 of the text. How many ways are there to arrange 7 elves and 5 goblins in a row in such a way that no goblins stand next to each other?

(4) Suppose we place any 5 points in a square that has side length equal to 2. Prove that two of the points must be within $\sqrt{2}$ distance of each other.

(5) Prove that if p is a prime number and $1 \leq k \leq p-1$ then p divides the binomial coefficient $\binom{p}{k}$.

(6) Problem 2, Section 2.1 of the text. Determine the number of ordered pairs (A, B), where $A \subseteq B \subseteq \{1, 2, \ldots, n\}$.

(Optional Review) Read sections 1.2(sets), 1.3(induction), and 1.4(functions) of your text.

(Optional Challenge) (a) (algebraic) Count the number of onto maps from a finite set of size m to a finite set of size n.
(b) (geometric) Number 24 of Section 2.3 of the text.